# **16.1** | Traveling Waves

# Learning Objectives

By the end of this section, you will be able to:

- · Describe the basic characteristics of wave motion
- Define the terms wavelength, amplitude, period, frequency, and wave speed
- Explain the difference between longitudinal and transverse waves, and give examples of each type
- List the different types of waves

We saw in **Oscillations** that oscillatory motion is an important type of behavior that can be used to model a wide range of physical phenomena. Oscillatory motion is also important because oscillations can generate waves, which are of fundamental importance in physics. Many of the terms and equations we studied in the chapter on oscillations apply equally well to wave motion (**Figure 16.2**).



**Figure 16.2** An ocean wave is probably the first picture that comes to mind when you hear the word "wave." Although this breaking wave, and ocean waves in general, have apparent similarities to the basic wave characteristics we will discuss, the mechanisms driving ocean waves are highly complex and beyond the scope of this chapter. It may seem natural, and even advantageous, to apply the concepts in this chapter to ocean waves, but ocean waves are nonlinear, and the simple models presented in this chapter do not fully explain them. (credit: Steve Jurvetson)

# **Types of Waves**

A **wave** is a disturbance that propagates, or moves from the place it was created. There are three basic types of waves: mechanical waves, electromagnetic waves, and matter waves.

Basic **mechanical waves** are governed by Newton's laws and require a medium. A medium is the substance a mechanical waves propagates through, and the medium produces an elastic restoring force when it is deformed. Mechanical waves transfer energy and momentum, without transferring mass. Some examples of mechanical waves are water waves, sound waves, and seismic waves. The medium for water waves is water; for sound waves, the medium is usually air. (Sound waves can travel in other media as well; we will look at that in more detail in **Sound**.) For surface water waves, the disturbance occurs on the surface of the water, perhaps created by a rock thrown into a pond or by a swimmer splashing the surface repeatedly. For sound waves, the disturbance is a change in air pressure, perhaps created by the oscillating cone

inside a speaker or a vibrating tuning fork. In both cases, the disturbance is the oscillation of the molecules of the fluid. In mechanical waves, energy and momentum transfer with the motion of the wave, whereas the mass oscillates around an equilibrium point. (We discuss this in **Energy and Power of a Wave**.) Earthquakes generate seismic waves from several types of disturbances, including the disturbance of Earth's surface and pressure disturbances under the surface. Seismic waves travel through the solids and liquids that form Earth. In this chapter, we focus on mechanical waves.

*Electromagnetic waves* are associated with oscillations in electric and magnetic fields and do not require a medium. Examples include gamma rays, X-rays, ultraviolet waves, visible light, infrared waves, microwaves, and radio waves. Electromagnetic waves can travel through a vacuum at the speed of light,  $v = c = 2.99792458 \times 10^8$  m/s. For example, light from distant stars travels through the vacuum of space and reaches Earth. Electromagnetic waves have some characteristics that are similar to mechanical waves; they are covered in more detail in **Electromagnetic Waves (http://cnx.org/content/m58495/latest/)**.

*Matter waves* are a central part of the branch of physics known as quantum mechanics. These waves are associated with protons, electrons, neutrons, and other fundamental particles found in nature. The theory that all types of matter have wavelike properties was first proposed by Louis de Broglie in 1924. Matter waves are discussed in **Photons and Matter Waves (http://cnx.org/content/m58757/latest/)**.

# **Mechanical Waves**

Mechanical waves exhibit characteristics common to all waves, such as amplitude, wavelength, period, frequency, and energy. All wave characteristics can be described by a small set of underlying principles.

The simplest mechanical waves repeat themselves for several cycles and are associated with simple harmonic motion. These simple harmonic waves can be modeled using some combination of sine and cosine functions. For example, consider the simplified surface water wave that moves across the surface of water as illustrated in **Figure 16.3**. Unlike complex ocean waves, in surface water waves, the medium, in this case water, moves vertically, oscillating up and down, whereas the disturbance of the wave moves horizontally through the medium. In **Figure 16.3**, the waves causes a seagull to move up and down in simple harmonic motion as the wave crests and troughs (peaks and valleys) pass under the bird. The crest is the highest point of the wave, and the trough is the lowest part of the wave. The time for one complete oscillation of the up-and-down motion is the wave's period *T*. The wave's frequency is the number of waves that pass through a point per unit time and is equal to f = 1/T. The period can be expressed using any convenient unit of time but is usually measured

in seconds; frequency is usually measured in hertz (Hz), where  $1 \text{ Hz} = 1 \text{ s}^{-1}$ .

The length of the wave is called the **wavelength** and is represented by the Greek letter lambda ( $\lambda$ ), which is measured in

any convenient unit of length, such as a centimeter or meter. The wavelength can be measured between any two similar points along the medium that have the same height and the same slope. In **Figure 16.3**, the wavelength is shown measured between two crests. As stated above, the period of the wave is equal to the time for one oscillation, but it is also equal to the time for one wavelength to pass through a point along the wave's path.

The amplitude of the wave (*A*) is a measure of the maximum displacement of the medium from its equilibrium position. In the figure, the equilibrium position is indicated by the dotted line, which is the height of the water if there were no waves moving through it. In this case, the wave is symmetrical, the crest of the wave is a distance +A above the equilibrium position, and the trough is a distance -A below the equilibrium position. The units for the amplitude can be centimeters or meters, or any convenient unit of distance.



**Figure 16.3** An idealized surface water wave passes under a seagull that bobs up and down in simple harmonic motion. The wave has a wavelength  $\lambda$ , which is the distance between adjacent identical parts of the wave. The amplitude *A* of the wave is the maximum displacement of the wave from the equilibrium position, which is indicated by the dotted line. In this example, the medium moves up and down, whereas the disturbance of the surface propagates parallel to the surface at a speed *v*.

The water wave in the figure moves through the medium with a propagation velocity  $\vec{v}$ . The magnitude of the **wave velocity** is the distance the wave travels in a given time, which is one wavelength in the time of one period, and the **wave speed** is the magnitude of wave velocity. In equation form, this is

$$v = \frac{\lambda}{T} = \lambda f.$$
 (16.1)

This fundamental relationship holds for all types of waves. For water waves, v is the speed of a surface wave; for sound, v is the speed of sound; and for visible light, v is the speed of light.

## Transverse and Longitudinal Waves

We have seen that a simple mechanical wave consists of a periodic disturbance that propagates from one place to another through a medium. In **Figure 16.4**(a), the wave propagates in the horizontal direction, whereas the medium is disturbed in the vertical direction. Such a wave is called a **transverse wave**. In a transverse wave, the wave may propagate in any direction, but the disturbance of the medium is perpendicular to the direction of propagation. In contrast, in a **longitudinal wave** or compressional wave, the disturbance is parallel to the direction of propagation. **Figure 16.4**(b) shows an example of a longitudinal wave. The size of the disturbance is its amplitude *A* and is completely independent of the speed of propagation *v*.



(a) Transverse wave

(b) Longitudinal wave

**Figure 16.4** (a) In a transverse wave, the medium oscillates perpendicular to the wave velocity. Here, the spring moves vertically up and down, while the wave propagates horizontally to the right. (b) In a longitudinal wave, the medium oscillates parallel to the propagation of the wave. In this case, the spring oscillates back and forth, while the wave propagates to the right.

A simple graphical representation of a section of the spring shown in **Figure 16.4**(b) is shown in **Figure 16.5**. **Figure 16.5**(a) shows the equilibrium position of the spring before any waves move down it. A point on the spring is marked with a blue dot. **Figure 16.5**(b) through (g) show snapshots of the spring taken one-quarter of a period apart, sometime after the end of` the spring is oscillated back and forth in the *x*-direction at a constant frequency. The disturbance of the wave is seen as the compressions and the expansions of the spring. Note that the blue dot oscillates around its equilibrium position a distance *A*, as the longitudinal wave moves in the positive *x*-direction with a constant speed. The distance *A* is the amplitude of the wave. The *y*-position of the dot does not change as the wave moves through the spring. The wavelength of the wave is measured in part (d). The wavelength depends on the speed of the wave and the frequency of the driving force.



**Figure 16.5** (a) This is a simple, graphical representation of a section of the stretched spring shown in **Figure 16.4**(b), representing the spring's equilibrium position before any waves are induced on the spring. A point on the spring is marked by a blue dot. (b–g) Longitudinal waves are created by oscillating the end of the spring (not shown) back and forth along the *x*-axis. The longitudinal wave, with a wavelength  $\lambda$ , moves along the spring in the +*x*-direction with a wave speed *v*. For convenience, the wavelength is measured in (d). Note that the point on the spring that was marked with the blue dot moves back and forth a distance *A* from the equilibrium position, oscillating around the equilibrium position of the point.

Waves may be transverse, longitudinal, or a combination of the two. Examples of transverse waves are the waves on stringed instruments or surface waves on water, such as ripples moving on a pond. Sound waves in air and water are longitudinal. With sound waves, the disturbances are periodic variations in pressure that are transmitted in fluids. Fluids do not have appreciable shear strength, and for this reason, the sound waves in them are longitudinal waves. Sound in solids can have both longitudinal and transverse components, such as those in a seismic wave. Earthquakes generate seismic waves under Earth's surface with both longitudinal and transverse components (called compressional or P-waves and shear or S-waves, respectively). The components of seismic waves have important individual characteristics—they propagate at different speeds, for example. Earthquakes also have surface waves that are similar to surface waves on water. Ocean waves also have both transverse and longitudinal components.

### Example 16.1

#### Wave on a String

A student takes a 30.00-m-long string and attaches one end to the wall in the physics lab. The student then holds the free end of the rope, keeping the tension constant in the rope. The student then begins to send waves down the string by moving the end of the string up and down with a frequency of 2.00 Hz. The maximum displacement of the end of the string is 20.00 cm. The first wave hits the lab wall 6.00 s after it was created. (a) What is the speed of the wave? (b) What is the period of the wave? (c) What is the wavelength of the wave?

#### Strategy

- a. The speed of the wave can be derived by dividing the distance traveled by the time.
- b. The period of the wave is the inverse of the frequency of the driving force.
- c. The wavelength can be found from the speed and the period  $v = \lambda/T$ .

#### Solution

a. The first wave traveled 30.00 m in 6.00 s:

$$v = \frac{30.00 \text{ m}}{6.00 \text{ s}} = 5.00 \frac{\text{m}}{\text{s}}$$

b. The period is equal to the inverse of the frequency:

$$T = \frac{1}{f} = \frac{1}{2.00 \text{ s}^{-1}} = 0.50 \text{ s}.$$

c. The wavelength is equal to the velocity times the period:

$$\lambda = vT = 5.00 \frac{\text{m}}{\text{s}} (0.50 \text{ s}) = 2.50 \text{ m}.$$

#### Significance

The frequency of the wave produced by an oscillating driving force is equal to the frequency of the driving force.

**16.1 Check Your Understanding** When a guitar string is plucked, the guitar string oscillates as a result of waves moving through the string. The vibrations of the string cause the air molecules to oscillate, forming sound waves. The frequency of the sound waves is equal to the frequency of the vibrating string. Is the wavelength of the sound wave always equal to the wavelength of the string?

#### Example 16.2

#### **Characteristics of a Wave**

A transverse mechanical wave propagates in the positive *x*-direction through a spring (as shown in **Figure 16.4**(a)) with a constant wave speed, and the medium oscillates between +A and -A around an equilibrium position. The graph in **Figure 16.6** shows the height of the spring (*y*) versus the position (*x*), where the *x*-axis points in the direction of propagation. The figure shows the height of the spring versus the *x*-position at t = 0.00 s as a dotted line and the wave at t = 3.00 s as a solid line. (a) Determine the wavelength and amplitude of the wave. (b) Find the propagation velocity of the wave. (c) Calculate the period and frequency of the wave.



#### Strategy

- a. The amplitude and wavelength can be determined from the graph.
- b. Since the velocity is constant, the velocity of the wave can be found by dividing the distance traveled by the wave by the time it took the wave to travel the distance.
- **c**. The period can be found from  $v = \frac{\lambda}{T}$  and the frequency from  $f = \frac{1}{T}$ .

#### Solution

a. Read the wavelength from the graph, looking at the purple arrow in **Figure 16.7**. Read the amplitude by looking at the green arrow. The wavelength is  $\lambda = 8.00$  cm and the amplitude is A = 6.00 cm.



b. The distance the wave traveled from time t = 0.00 s to time t = 3.00 s can be seen in the graph. Consider the red arrow, which shows the distance the crest has moved in 3 s. The distance is 8.00 cm - 2.00 cm = 6.00 cm. The velocity is

$$v = \frac{\Delta x}{\Delta t} = \frac{8.00 \text{ cm} - 2.00 \text{ cm}}{3.00 \text{ s} - 0.00 \text{ s}} = 2.00 \text{ cm/s}.$$
  
c. The period is  $T = \frac{\lambda}{v} = \frac{8.00 \text{ cm}}{2.00 \text{ cm/s}} = 4.00 \text{ s}$  and the frequency is  $f = \frac{1}{T} = \frac{1}{4.00 \text{ s}} = 0.25 \text{ Hz}.$ 

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#### Significance

Note that the wavelength can be found using any two successive identical points that repeat, having the same

height and slope. You should choose two points that are most convenient. The displacement can also be found using any convenient point.

**16.2** Check Your Understanding The propagation velocity of a transverse or longitudinal mechanical wave may be constant as the wave disturbance moves through the medium. Consider a transverse mechanical wave: Is the velocity of the medium also constant?

# **16.2** Mathematics of Waves

# Learning Objectives

By the end of this section, you will be able to:

- Model a wave, moving with a constant wave velocity, with a mathematical expression
- Calculate the velocity and acceleration of the medium
- Show how the velocity of the medium differs from the wave velocity (propagation velocity)

In the previous section, we described periodic waves by their characteristics of wavelength, period, amplitude, and wave speed of the wave. Waves can also be described by the motion of the particles of the medium through which the waves move. The position of particles of the medium can be mathematically modeled as **wave functions**, which can be used to find the position, velocity, and acceleration of the particles of the medium of the wave at any time.

## Pulses

A **pulse** can be described as wave consisting of a single disturbance that moves through the medium with a constant amplitude. The pulse moves as a pattern that maintains its shape as it propagates with a constant wave speed. Because the wave speed is constant, the distance the pulse moves in a time  $\Delta t$  is equal to  $\Delta x = v\Delta t$  (Figure 16.8).



amplitude *A*. The pulse moves as a pattern with a constant shape, with a constant maximum value *A*. The velocity is constant and the pulse moves a distance  $\Delta x = v\Delta t$  in a time  $\Delta t$ . The distance traveled is measured with any convenient point on the pulse. In this figure, the crest is used.

# Modeling a One-Dimensional Sinusoidal Wave using a Wave Function

Consider a string kept at a constant tension  $F_T$  where one end is fixed and the free end is oscillated between y = +A and